Near-Feasible Stable Matchings: Incentives and Optimality

Extended Abstract

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ABSTRACT

Stable matching is a fundamental area with many practical applications. Recent work has introduced the paradigm of near-feasibility in capacitated matching settings, where agent capacities are slightly modified to ensure the existence of desirable outcomes. While useful when no stable matching exists or when some agents are left unmatched otherwise, it has not previously been investigated whether near-feasible stable matchings satisfy desirable properties with respect to their stability in the original instance. Furthermore, prior work leaves open the deviation incentive issues that arise when the centralised authority modifies agents' capacities. We consider these issues in the Stable Fixtures problem model, which generalises many classical models through non-bipartite preferences and capacitated agents. We develop a formal framework combining near-feasibility and almost-stability to analyse and quantify agent incentives to adhere to computed matchings. We study the trade-offs between instability, capacity modifications, and computational complexity. Further, we show that different modification strategies significantly affect stability, but establish that minimal modifications and minimal deviation incentives are compatible and efficiently computable.

KEYWORDS

Stable Matching; Near-Feasibility; Almost-Stability; Stable Fixtures

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1 INTRODUCTION

Stable matching theory lies at the heart of centralised matching and allocation mechanisms for multi-agent systems [32]. The classical Stable Marriage problem deals with two equally-sized and non-overlapping sets of agents (which we will refer to as a *bipartition*) and their strict ordinal preference relations over agents from the set that they are not a part of (i.e., their potential partners). A solution to the problem is a matching free of blocking pairs. This is important, as a blocking pair gives agents an incentive for decentralised coordination and deviation, which might cause unravelling. We refer to a matching free of blocking pairs as *stable* and the instance as *solvable*. Gale and Shapley showed that, in this model, a stable matching always exists and can be found efficiently [20]. Two important extensions of the Stable Marriage problem involve the existence of *capacitated agents*, i.e., agents that can accommodate more than one partner, and the possibility of *non-bipartite preferences*, i.e., the absence of a bipartition among agents. The Stable Fixtures problem (sf) introduced by Irving and Scott [31] permits both of these extensions simultaneously.

*Definition 1.1 (*sf *Instance).* Let $I = (A, \succ, c)$ be an sf *instance*, where $A = \{a_1, a_2, \dots, a_n\}$, also denoted by $A(I)$, is a set of $n \in \mathbb{N}$ agents, $\succ$ is a tuple of n strict ordinal *preference rankings* $\succ_i$ for each agent a_i over all other agents $A \setminus \{a_i\}$, and $c : A \rightarrow \{1, 2, \dots, n-1\}$ is a capacity function indicating the maximum number c_i of agents that each agent $a_i \in A$ can be matched to.

A matching in an sf instance I consists of unordered pairs such that no agent's capacity is exceeded. We say that a matching M is *stable* if it does not admit blocking pairs, where a *blocking pair* of M is formally defined as distinct agents $a_i, a_j \in A$ such that

(1) $\{a_i, a_j\} \notin M$, and
(2) $|M(a_i)| < c_i$ or $a_j \succ_i \text{worst}_i(M(a_i))$, and
(3) $|M(a_j)| < c_j$ or $a_i \succ_j \text{worst}_j(M(a_j))$,

where $\text{worst}_i(M(a_i))$ denote the worst agent assigned to a_i in M according to $\succ_i$. This general model captures a wide range of real-world multi-agent interactions: from assigning students to projects and residents to hospitals, to pairing competitors in tournaments and participants in kidney exchanges [32]. However, unlike bipartite matching settings, these environments lack desirable structural guarantees: stable outcomes may not exist [29, 31]. This motivates the search for alternative solution concepts that maintain minimal deviation incentives and desirable stability properties in practice.

Two prevalent directions for alternative solutions in stable matching more broadly are *almost-stability* and *near-feasibility*, where the aim with the former is to find a matching with a minimum number of blocking pairs (or a minimum number of blocking agents or a minimax number of blocking pairs per agent) [1, 7, 14, 23], to minimise the likelihood of the matching being undermined [18, 32]. The aim with near-feasibility is to find a new instance in which few agents' capacities are modified in such a way that the resulting new instance guarantees the existence of one or more stable matchings [35]. From a multi-agent perspective, these notions reflect two distinct approaches to managing instability: either tolerate limited incentives to deviate (almost-stability), or minimally adjust system parameters (capacities) to restore collective stability (near-feasibility). Unfortunately, most problems involving the computation of almost-stable

The full version of this paper can be found at reference [21].

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matchings do not admit efficient algorithms, even in very restricted settings [1, 5, 6, 12, 14, 24, 25, 28, 30, 33]. On the other hand, there is a very active line of research that investigates the tractability frontier and the design of efficient algorithms for problems involving notions of near-feasibility [11, 13, 15–17, 22, 26, 27, 34, 36, 37] and the related problem of *capacity planning* [2, 3, 8]. However, one major shortcoming of near-feasible stable matchings is that they do not necessarily have desirable stability properties with respect to the original instance (without modified capacities). In particular, in a near-feasible stable matching, agents might end up in strictly fewer or strictly more pairs than desired, leading to an inherent instability.

We discuss related work in more detail in the full version of this paper [21], and point out references [4, 9, 10, 19] for further important results on SF and related general matching models.

2 OUR CONTRIBUTIONS

We study the incentive and complexity landscape underlying near-feasible stable matchings in multi-agent systems where agents can form multiple partnerships. Within the Stable Fixtures model, we investigate the trade-off between system-level feasibility and individual-level stability, and develop new methods to quantify and manage agent incentives under capacity modifications. Our main contributions are fourfold:

(1) We introduce new concepts and problem models that connect near-feasibility with explicit agent-level incentive measures.
(2) We develop efficient algorithms for finding matchings and capacity modifications that balance feasibility and stability.
(3) We derive tight bounds and structural characterisations for these settings, highlighting the interplay between incentives and modifications at the individual and aggregate levels.
(4) We show through an empirical investigation that the minimum number of capacity modifications required to arrive at solvability and the minimum instability are often small.

Beyond theoretical interest, our results contribute to the broader goal of designing fair, desirable, and scalable coordination mechanisms for multi-agent environments.

More formally, we introduce the following notation. We refer to a new capacity function c' as a *Minimal Individual (respectively Aggregate) Modification*, denoted MIM (MAM), if it minimises the maximum (total) number of capacity changes required to arrive at a solvable instance. Variants with subscript $\pm$ indicate that capacity increases and decreases are permitted, while $+$ variants permit only increases. We prove that there always exists a c' that is all of $\text{MIM}_\pm$, MIM_+, $\text{MAM}_\pm$, and MAM_+ simultaneously, and show how to compute one in quadratic time. Notably, no agent's capacity needs to be changed by more than 1 in c', although the total number of changes is $\Theta(n)$ in the worst case, where n is the number of agents.

Furthermore, we quantify instability through a new notion of *blocking entries* that generalises blocking pairs.

Definition 2.1 (Blocking Entry). Let $I = (A, \succ, c)$ be an SF instance, let c' be an alternative capacity function for I, and let M be a matching in $I' = (A, \succ, c')$. Then, for two distinct agents $a_i, a_j \in A$, a_j is a *blocking entry* of a_i, denoted by an ordered tuple (a_i, a_j), if

(1) $\{a_i, a_j\} \in M$ and $|\{a_r \in M(a_i) \mid a_r \succ_i a_j\}| \geq c_i$, or
(2) $\{a_i, a_j\} \notin M$, $|\{a_r \in M(a_i) \mid a_r \succ_i a_j\}| < c_i$ and $|\{a_r \in M(a_j) \mid a_r \succ_j a_i\}| < c_j$.

Let $be_i(M)$ denote the set of blocking entries of a_i admitted by M and let $be(M) = \bigcup_{a_i \in A} be_i(M)$.

We refer to a matching M as having *Minimal Individual (respectively Aggregate) Deviation Incentive* with respect to an original instance I and a set of alternative capacity functions C, denoted MIDI_C (MADI_C), if, for all $c' \in C$ and for all matchings M' of $I' = (A, \succ, c')$, it is the case that $|be(M)| \leq \max_{a_i \in A} |be_i(M')|$ (or $|be(M)| \leq |be(M')|$), where the blocking entries are measured against I. Informally, MIDI matchings minimise the maximum incentive of any individual agent to deviate among all feasible matchings, where feasibility is controlled through a collection of capacity functions C. MADI matchings minimise the total incentive to deviate, aiming to stabilise the system as a whole.

Our algorithmic results for the computation of such matchings for various possibilities for C show that when no capacity violations are permitted, the problem of computing a MIDI matching is para-NP-hard with respect to the optimal maximum number of blocking entries, and the problem of computing a MADI matching is NP-hard but in XP with respect to the optimal total number of blocking entries. On the other hand, when not putting any restrictions on the alternative capacity functions, we show that both problems are in P. Furthermore, we combine these notions with MIM and MAM to show that a matching that simultaneously minimises the maximum and total number of blocking pairs with respect to the original instance, while also not requiring capacity modification worse than the optimal capacity modifications for finding a near-feasible solvable instance, can always be computed efficiently.

These positive results show that optimality on the individual and aggregate levels with regards to minimum deviation incentives is compatible and – surprisingly – that a matching satisfying both criteria simultaneously can be computed efficiently. In [21], we also prove further compatibility and incompatibility results, provide exact exponential-time algorithms for the (para-)NP-hard problem variants, and provide experimental results for instances with preferences generated uniformly at random.

3 CONCLUSION

We studied the deviation incentives and complexity of near-feasible stable matchings and their computation in the non-bipartite, capacitated stable matching setting. Several intriguing directions remain. We introduced new optimality measures for matchings in the many-to-many setting; it would be interesting to evaluate how other existing approaches in many-to-one and many-to-many models perform under these criteria, and whether certain techniques systematically minimise agents' incentives to deviate. From an economic perspective, analysing real-world matching markets through the lens of blocking entries and deviation incentives could yield interesting insights. Algorithmically, one could study special cases where $\text{MIDI}_{\{c\}}$ and $\text{MADI}_{\{c\}}$ matchings can be computed efficiently.

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