

Non-Bipartite Stable Matching and Beyond

Doctoral Consortium

Frederik Glitzner
 School of Computing Science
 University of Glasgow
 Glasgow, United Kingdom
 f.glitzner.1@research.gla.ac.uk

ABSTRACT

Non-bipartite matching problems arise naturally in many multi-agent systems, yet they face a fundamental challenge absent from classical bipartite markets: even under strict preferences, stable matchings need not exist. This non-existence requires a careful combinatorial study of preference structures, and calls for the design of advanced algorithms when aiming for alternative fairness or relaxed stability guarantees in such settings. I study the structural origins of instability in non-bipartite preference systems, develop new optimality criteria inspired by both classical social choice theory and by real-world requirements, and design efficient algorithms for the computation of matchings satisfying such criteria if possible.

KEYWORDS

Stable matching; Non-bipartite matching; Fairness; Algorithmics

ACM Reference Format:

Frederik Glitzner. 2026. Non-Bipartite Stable Matching and Beyond: Doctoral Consortium. In *Proc. of the 25th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2026), Paphos, Cyprus, May 25 – 29, 2026*, IFAAMAS, 3 pages. <https://doi.org/10.65109/SLTP2592>

1 INTRODUCTION

Centralised matching schemes play a key role in many real-world matching markets, such as organ donation or exchange programs, team formation, and specialised job markets. In practice, trade-offs often need to be made between the size, fairness, and efficient computation of desirable matchings [32]. Furthermore, the incentives of participating agents, and thus the strategic implications for the matching criteria, need to be considered. One criterion that is often used in practice is *stability*, that is, the absence of two agents who could be better off by being matched to each other instead of one of their allocated partners (or they have free capacity) [14]. Stable matching theory combines foundational concepts from economics (such as cooperative game theory and fairness), mathematics (such as graph theory and discrete optimisation), and computer science (such as efficient algorithm design and computational complexity) [32]. Much progress has been made in characterising the structure of stable matchings and classifying the complexity of practically-motivated optimisation problems, usually in domains where agents or resources of one type are matched to agents or resources of

a different type (e.g., workers to firms or houses to applicants) – giving rise to *bipartite* matching problems [32]. In contrast, *non-bipartite* matching problems involving the allocation of agents or resources of the same type to each other (e.g., partners to each other, donor-recipient pairs to each other, or other peer-to-peer markets) are often much less well-behaved, exhibiting a more challenging structure, and can lead to computationally harder problems [27, 32, 34]. These weaker guarantees and stronger challenges often require a more sophisticated design and analysis, as well as novel solution concepts. Thus, in the next sections, we will discuss gaps, new results, and ongoing challenges concerning such *non-bipartite* (stable) matchings, starting with the one-to-one setting (in which every agent can have at most one partner), and continuing with the many-to-many setting, where agents have an integral capacity indicating the maximum number of partners that they can have. We conclude with a thesis plan and future work.

2 ONE-TO-ONE MATCHING

Algorithmics and Structure of Stability. The fundamental limitation in the non-bipartite setting was identified already in the seminal work by Gale and Shapley [14]: even under strict complete preference lists, a stable matching might not exist. On the positive side, Irving [29] provided an efficient algorithm to find a stable matching, if one exists, and the structure of stable matchings has been characterised extensively [26, 27, 30, 32]. Tan [36, 37], on the other hand, characterised the preference structures leading to the *non-existence* of stable matchings through a combinatorial structure that he referred to as a *stable partition*. In Glitzner and Manlove [17, 24], we provide new structural and algorithmic results for stable partitions, further characterising stable and “unstable” preference structures, and providing a better understanding of fair stable half-integral matchings [13]. Furthermore, in Glitzner and Manlove [18], we provide new insights on the likelihood and size of unstable preference structures, as well as resulting implications for various other key solution concepts. Based on comprehensive simulations, we conclude, for example, that removing one percent of the agents from the instance typically leads to the existence of stable matchings.

Almost-Stability and Beyond. When forced to accept some instability, existing work on so-called *almost-stable matchings* has largely focused on aggregate instability measures that, for example, minimise the total number of blocking pairs or minimise the number of agents involved in at least one blocking pair [3, 32]. Unfortunately, most such optimisation problems turn out to be highly computationally intractable [1, 4, 5]. Eriksson and Häggström [11] argued that counting blocking pairs is the most indicative measure



This work is licensed under a Creative Commons Attribution International 4.0 License.

Proc. of the 25th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2026), C. Amato, L. Dennis, V. Mascardi, J. Thangarajah (eds.), May 25 – 29, 2026, Paphos, Cyprus. © 2026 International Foundation for Autonomous Agents and Multiagent Systems (www.ifaamas.org). <https://doi.org/10.65109/SLTP2592>

of instability, as each such pair has both the incentive and the opportunity to deviate. However, in Glitzner and Manlove [21], we argue that aggregate approaches can permit large concentrated instability on a few individuals. Thus, both from the perspective of individual fairness, as well as from a robustness perspective, this appears problematic: if each blocking pair represents a legitimate injustice and causes a realistic potential for deviations and unravelling, why should we allow blocking pairs to accumulate in a few individuals? We introduce and study a new *minimax* notion of almost-stability, which requires that the maximum number of blocking pairs that any agent finds themselves in is minimal. We establish that even deciding the existence of a matching in which no agent is in more than one blocking pair is NP-complete, although we provide efficient algorithms for the case where preference lists are very short. In follow-up work [22], we also provide an efficient approximation algorithm and integer linear programming-based algorithms for this problem, and establish experimentally that a well-balanced (from a minimax perspective) almost-stable matching is likely to exist for instances with uniformly random preferences.

In Glitzner and Manlove [21] (and follow-up work in [23]), we also consider another new perspective: say that we know that only a specific subset D of *deviator* agents can “act” on blocking pairs and initiate pairwise advantageous deviations. Can we decide efficiently whether there exists a matching in which no agent in D has an incentive to initiate such a deviation? We prove that the answer is no (unless $P=NP$), even under strong restrictions. However, we provide parametrised algorithms and polynomial-time algorithms for instances with very short lists, and study related problems.

3 MANY-TO-MANY MATCHING

Stability and Near-Feasibility. Under strict preferences, the efficient algorithm to find a stable matching (if one exists) was extended to the many-to-many setting by Irving and Scott [31]. Furthermore, Fleiner [13] showed how the many-to-many problem can be reduced to the one-to-one problem. While a great deal of structure is known for the one-to-one setting, surprisingly little has been shown for the many-to-many generalisation. In Glitzner and Manlove [19, 20], we define stable partitions in this more general setting, tightly characterise stable and unstable preference structures, and develop an algorithm that modifies a few agents’ capacities to arrive at an instance that admits stable matchings. This fits into an important new paradigm of *near-feasible* stable matchings, which had previously been studied in other settings [9, 33].

Unifying Perspectives. In Glitzner [15, 16], I embed this new paradigm of near-feasibility into a formal framework that unifies almost-stability (which gives rise to notoriously intractable problems) and near-feasibility (which has led to many efficient algorithms [7, 10, 20]), and study the compatibility and complexity of finding optimal capacity modifications across individual and aggregate criteria. Surprisingly, unlike the computation of almost-stable matchings, where capacity modifications are not permitted, finding a matching that simultaneously minimises capacity extensions and agent incentives to deviate on both individual and aggregate levels turns out to be polynomial-time solvable. Furthermore, we show experimentally that, on average, only very few capacity modifications are necessary to arrive at an instance that admits stable matchings.

4 CONCLUSION AND FUTURE WORK

Overall, non-bipartite stable matching problems suffer from a major limitation: stable matchings do not always exist. Yet, non-bipartite matching markets play a crucial role in today’s world, where peer-to-peer problems are becoming increasingly common. In my thesis, I plan to delineate the inherent tradeoffs between economically desirable solutions and the computational complexity of finding, verifying, and manipulating them, in general multi-agent matching models, to answer: how should we design, compute, and evaluate matching algorithms when stability is unattainable or insufficient?

For the remainder of my PhD, I plan to continue studying matching under preferences through algorithmic and combinatorial theory, as well as experimentation. I would like to investigate what can be learned from non-bipartite (stable) matching theory for more general multi-agent problems and computational social choice, and vice versa. I will illustrate this with two ongoing research directions.

Justified Envy-Freeness. A key fairness criterion in fair division is that of *envy-freeness*, or approximate versions thereof [35]. In bipartite matching with two-sided preferences, a weakened notion of *justified envy-freeness* can be considered. In the one-to-one setting, stability and justified envy-freeness (together with non-wastefulness) are equivalent [2]. However, the same need does not hold for many-to-many matching settings, and, in fact, when ties are permitted, then (under certain preference rules) a justified envy-free matching might not exist at all. Still, approximately justified envy-free matchings do, as Igarashi et al. [28] recently showed.

These new perspectives for the bipartite setting also motivate the study of justified envy-freeness in the non-bipartite many-to-many matching setting. Our preliminary results show that, under strict preferences, any stable matching is justified envy-free, but the converse need not hold. This is positive: although these instances might not admit stable matchings, they can still admit justified envy-free matchings. Now, can we decide the existence of a justified envy-free matching in polynomial time? What approximations of justified envy-freeness are guaranteed to exist?

Popular Matchings. Another well-studied alternative to stability is *popularity*. Informally, a matching is popular if it does not lose in a head-to-head election against any other matching, where agents’ votes depend only on their own partners [32]. Equivalently, popular matchings are weak Condorcet winners [6]. It is well-known that, under strict preferences, any stable matching is popular, although the converse is not true in general. Similar to justified envy-freeness, this means that a popular matching might exist even when stable matchings do not. However, it is NP-complete to decide the existence of popular matchings in STABLE ROOMMATES [12, 25]. In Csáji and Glitzner [8], we show in the bipartite setting that popular matchings can give significant increases in matching size compared to stable matchings, without introducing much instability. It would be interesting to develop fast exponential-time or parametrised algorithms to extend this study to the non-bipartite setting.

ACKNOWLEDGMENTS

I would like to thank my supervisor, Professor David Manlove, for his invaluable support and guidance, and the School of Computing Science at the University of Glasgow for my Minerva Scholarship.

REFERENCES

[1] David J. Abraham, Péter Biro, and David Manlove. 2006. “Almost stable” matchings in the roommates problem. *Proceedings of WAOA 2005* 3879 (2006), 1–14. https://doi.org/10.1007/11671411_1

[2] Michel Balinski and Tayfun Sönmez. 1999. A Tale of Two Mechanisms: Student Placement. *Journal of Economic Theory* 84, 1 (1999), 73–94. <https://doi.org/10.1006/jeth.1998.2469>

[3] Jiehua Chen. 2019. Computational Complexity of Stable Marriage and Stable Roommates and Their Variants. arXiv:1904.08196 [cs.GT]

[4] Jiehua Chen, Danny Hermelin, Manuel Sorge, and Harel Yedidsion. 2018. How hard is it to satisfy (almost) all roommates? *Proceedings of ICALP 2018* 107 (7 2018), 35:1–35:15. <https://doi.org/10.4230/LIPICs.ICALP.2018.35>

[5] Jiehua Chen, Sanjukta Roy, and Sofia Simola. 2025. FPT-Approximability of Stable Matching Problems. arXiv:2508.10129 [cs.GT] <https://arxiv.org/abs/2508.10129>

[6] Agnes Cseh and Telikepalli Kavitha. 2021. Popular Matchings in Complete Graphs. *Algorithmica* 83 (5 2021), 1493–1523. <https://doi.org/10.1007/s00453-020-00791-7>

[7] Gergely Csáji. 2025. Near-Feasible Solutions to Complex Stable Matching Problems. arXiv:2502.02503 [cs.GT] <http://arxiv.org/abs/2502.02503>

[8] Gergely Csáji and Frederik Glitzner. 2026. Weakly-Popular and Super-Popular Matchings with Ties and Their Connection to Stable Matchings. arXiv:2310.12269 [cs.GT] <https://arxiv.org/abs/2310.12269>

[9] Gergely Csáji, David Manlove, Iain McBride, and James Trimble. 2024. Couples can be tractable: New algorithms and hardness results for the Hospitals / Residents problem with Couples. arXiv:2311.00405 [cs.DS] <https://arxiv.org/abs/2311.00405>

[10] Gergely Csáji, David Manlove, Iain McBride, and James Trimble. 2024. Couples Can Be Tractable: New Algorithms and Hardness Results for the Hospitals/Residents Problem with Couples. In *Proceedings of IJCAI 2024*, Kate Larson (Ed.). IJCAI, Jeju, South Korea, 2731–2739. <https://doi.org/10.24963/ijcai.2024/302>

[11] Kimmo Eriksson and Olle Häggström. 2008. Instability of Matchings in Decentralized Markets with Various Preference Structures. *International Journal of Game Theory* 36, 3 (March 2008), 409–420. <https://doi.org/10.1007/s00182-007-0081-6>

[12] Yuri Faenza, Telikepalli Kavitha, Vladlena Powers, and Xingyu Zhang. 2019. Popular matchings and limits to tractability. In *Proceedings of the Thirtieth Annual ACM-SIAM Symposium on Discrete Algorithms* (San Diego, California) (SODA ’19). Society for Industrial and Applied Mathematics, USA, 2790–2809.

[13] Tamás Fleiner. 2008. Stable matchings through fixed points and graphs. *Annales Universitatis Scientiarum Budapestinensis de Rolando Eotvos Nominatae, Sectio Mathematica* 51 (2008), 69–116.

[14] David Gale and Lloyd S. Shapley. 1962. College Admissions and the Stability of Marriage. *The American Mathematical Monthly* 69 (1 1962), 9. Issue 1. <https://doi.org/10.2307/2312726>

[15] Frederik Glitzner. 2026. Near-Feasible Stable Matchings: Incentives and Optimality. arXiv:2602.10851 [cs.GT] <https://arxiv.org/abs/2602.10851>

[16] Frederik Glitzner. 2026. Near-Feasible Stable Matchings: Incentives and Optimality. In *Proceedings of AAMAS 2026*. IFAAMAS, Paphos, Cyprus, 3. <https://doi.org/10.65109/RZEM3915>

[17] Frederik Glitzner and David Manlove. 2024. Structural and Algorithmic Results for Stable Cycles and Partitions in the Roommates Problem. In *Proceedings of SAGT 2024*. Springer Nature Switzerland, Cham, Switzerland, 3–20. https://doi.org/10.1007/978-3-031-71033-9_1

[18] Frederik Glitzner and David Manlove. 2025. Perspectives on Unsolvability in the Roommates Problem. arXiv:2505.06717 [cs.GT] <https://arxiv.org/abs/2505.06717>

[19] Frederik Glitzner and David Manlove. 2025. Unsolvability and Beyond in Many-to-Many Non-bipartite Stable Matching. arXiv:2505.11456 [cs.DS] <https://arxiv.org/abs/2505.11456>

[20] Frederik Glitzner and David Manlove. 2025. Unsolvability and Beyond in Many-to-Many Non-bipartite Stable Matching. In *Proceedings of SAGT 2025*. Springer Nature Switzerland, Cham, Switzerland, 267–285. https://doi.org/10.1007/978-3-032-03639-1_15

[21] Frederik Glitzner and David Manlove. 2026. Minimax and Preferential Almost-Stable Matchings. In *Proceedings of AAMAS 2026*. IFAAMAS, Paphos, Cyprus, 9. <https://doi.org/10.65109/PCDE6577>

[22] Frederik Glitzner and David Manlove. 2026. A Minimax Perspective on Almost-Stable Matchings. arXiv:2601.14195 [cs.GT] <https://arxiv.org/abs/2601.14195>

[23] Frederik Glitzner and David Manlove. 2026. Stable Matching with Deviators and Conformists. arXiv:2601.18573 [cs.GT] <https://arxiv.org/abs/2601.18573>

[24] Frederik Glitzner and David Manlove. 2026. Structural and Algorithmic Results for Stable Cycles and Partitions in the Roommates Problem. *ACM Transactions on Economics and Computation* (1 2026). <https://doi.org/10.1145/3789257>

[25] Sushmita Gupta, Pranabendu Misra, Saket Saurabh, and Meirav Zehavi. 2021. Popular Matching in Roommates Setting Is NP-hard. *ACM Transactions on Computation Theory (TOCT)* 13 (3 2021), 1–20. Issue 2. <https://doi.org/10.1145/3442354>

[26] Dan Gusfield. 1988. The Structure of the Stable Roommate Problem: Efficient Representation and Enumeration of All Stable Assignments. *SIAM J. Comput.* 17 (7 1988), 742–769. Issue 4. <https://doi.org/10.1137/0217048>

[27] Daniel Gusfield and Robert Irving. 1989. *The Stable Marriage problem: Structure and Algorithms*. MIT Press, Cambridge, MA, USA.

[28] Ayumi Igarashi, Naoyuki Kamiyama, Yasushi Kawase, Warut Suksompong, Hanna Sumita, and Yu Yokoi. 2025. Two-Sided Fairness in Many-to-One Matching. In *Proceedings of WINE 2025*. arXiv Preprint, New Brunswick, United States. arXiv:2509.24111 [econ.TH] <https://arxiv.org/abs/2509.24111>

[29] Robert W. Irving. 1985. An efficient algorithm for the “stable roommates” problem. *Journal of Algorithms* 6 (12 1985), 577–595. Issue 4. [https://doi.org/10.1016/0196-6774\(85\)90033-1](https://doi.org/10.1016/0196-6774(85)90033-1)

[30] Robert W. Irving. 1986. *On the stable roommates problem*. Department of Computing Science, University of Glasgow.

[31] Robert W. Irving and Sandy Scott. 2007. The stable fixtures problem—A many-to-many extension of stable roommates. *Discrete Applied Mathematics* 155 (10 2007), 2118–2129. Issue 16. <https://doi.org/10.1016/J.DAM.2007.05.015>

[32] David Manlove. 2013. *Algorithmics of Matching Under Preferences*. Series on Theoretical Computer Science, Vol. 2. World Scientific, Singapore. <https://doi.org/10.1142/8591>

[33] Thành Nguyen and Rakesh Vohra. 2018. Near-Feasible Stable Matchings with Couples. *American Economic Review* 108, 11 (2018), 3154–3169. <https://doi.org/10.1257/aer.20141188>

[34] Eytan Ronn. 1990. NP-complete stable matching problems. *Journal of Algorithms* 11 (6 1990), 285–304. Issue 2. [https://doi.org/10.1016/0196-6774\(90\)90007-2](https://doi.org/10.1016/0196-6774(90)90007-2)

[35] Warut Suksompong. 2021. Constraints in Fair Division. *ACM SIGecom Exchanges* 19, 2 (2021), 46–61. <https://doi.org/10.1145/3505156.3505162>

[36] Jimmy J.M. Tan. 1991. A necessary and sufficient condition for the existence of a complete stable matching. *Journal of Algorithms* 12 (3 1991), 154–178. Issue 1. [https://doi.org/10.1016/0196-6774\(91\)90028-W](https://doi.org/10.1016/0196-6774(91)90028-W)

[37] Jimmy J.M. Tan. 1991. Stable matchings and stable partitions. *International Journal of Computer Mathematics* 39 (1 1991), 11–20. Issue 1-2. <https://doi.org/10.1080/00207169108803975>