

Project Submission Games in Participatory Budgeting

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ABSTRACT

We introduce the framework of *project submission games*, capturing the behavior of project proposers in participatory budgeting (and multiwinner elections). Here, each proposer submits a subset of project proposals, aiming at maximizing the total cost of those that get funded. We focus on finding conditions under which pure Nash equilibria exist in our games, and on the complexity of checking whether they exist. We also seek algorithms for computing best responses for the proposers. Finally, we check the existence of equilibria, as well as their properties, in real-life data.

KEYWORDS

Participatory Budgeting, Strategic Nomination, Equilibria

ACM Reference Format:

Piotr Faliszewski, Łukasz Janeczko, Andrzej Kaczmarczyk, Grzegorz Lisowski, and Grzegorz Pierczyński. 2026. Project Submission Games in Participatory Budgeting. In *Proc. of the 25th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2026)*, Paphos, Cyprus, May 25 – 29, 2026, IFAAMAS, 9 pages. <https://doi.org/10.65109/TZVZ6294>

1 INTRODUCTION

In a typical *participatory budgeting* (PB) scenario [6, 17, 32] a city first fixes some budget, then groups of activists propose projects that they would like to see implemented—each with its individual cost and scope—and, finally, the citizens vote on which of them should be carried out. We focus on the approval setting where the voters indicate for each project whether they support it or not, and we study the strategic issues that project proposers face when deciding which projects to submit.

Let us consider the following example. The city fixed the budget of €75 000 and two activist groups came up with, altogether, six projects that they could propose, diversified both in terms of their cost and in terms of their support. Specifically, Transparent Revolution in Environmental Engineering group (TREE) developed projects T_1 , T_2 , and T_3 , focused on building parks in different areas of the city, whereas the Bicycle Initiative and Kinetic Enthusiasts

society (BIKE) invented projects B_1 , B_2 , and B_3 , devoted to improving the cycling infrastructure. The costs and numbers of voters supporting each of them are given in Table 1.

The city decided to use the BasicAV voting rule,¹ which considers the projects from the most to the least supported one and selects those that fit into the remaining budget at the time of consideration. Hence, if both groups submitted all their projects, the rule would select T_1 , B_1 , B_2 , and B_3 . However, in order to have as much of the budget dedicated to their projects as possible, TREE did not submit T_1 , as then projects T_2 , B_1 , and T_3 would be selected, giving them €60 000 for their projects, instead of €50 000. However, BIKE realized that this might happen and chose not to submit B_1 . Consequently, projects T_2 , T_3 , B_2 and B_3 were funded. This was fortunate as neither of the groups could have improved further from this state. Hence, choosing what projects to submit involves non-trivial strategic considerations.

Table 1: Costs and levels of support of projects considered by TREE and BIKE groups.

	Cost (€)	Support		Cost (€)	Support
1 : T_1	50 000	6 000	4 : T_3	30 000	3 000
2 : T_2	30 000	5 000	5 : B_2	7 000	2 000
3 : B_1	10 000	4 000	6 : B_3	7 000	1 000

Our goal is to provide game-theoretic and computational analysis of scenarios analogous to those in the above example. To this end, we introduce *project submission games* (PSGs) where project proposers are the players, each proposer owns a set of projects, and their goal is to maximize the amount of money spent on their projects by choosing which ones to submit. We ask if our games always admit Nash equilibria and, if not, what is the complexity of deciding if they exist. We adopt the following assumptions:

- (1) The proposers have full knowledge of the setting. In particular, they know exactly which voters approve which projects. Naturally, this is a simplification—in practice, proposers have at most approximate information about voters’ preferences, e.g., via polls. In the theoretical sections, we believe that this restriction is justified, as a first necessary step.

¹We note that this rule is also called GreedyAV [2] or GreedCost [32]. We chose to call it BasicAV to emphasize that it is the rule most commonly used in practice.



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We further weaken this assumption in the experimental section (Section 6.3). In particular, we show that in realistic instances limited information regarding voters’ ballots has little effect on the proposers’ utility.

- (2) The voters approve projects independently from the presence of the other projects. This is natural in the context of approval-based elections, where voters can simply vote for the projects they consider worth implementing (this is in sync with so-called *threshold approval voting* [1, 11]).

We consider several variants of our games, depending on the costs of the projects, allowed project-submission strategies, and the structure of the preferences:

Costs. By default, we allow arbitrary project costs, but we also analyze the setting where all projects have equal, unit costs. This models the multiwinner setting [16, 21], where instead of deciding which projects to fund, the goal is to select a committee of a given, fixed size that represents the voters. Hence, in this case, the budget is the size of the committee and the “projects” are the candidates that run for the committee (yet, for consistency, we will use terms “budget” and “projects” even in this setting).

Submission Strategies. By default, each proposer can submit any nonempty subset of their projects, but we also consider the setting where each proposer submits exactly one project. This captures the scenario where the projects are variants of the same idea (e.g., with different scope, costs, and targeted for different voter groups).

Preference Structure. By default, we do not enforce any structure over the voters’ approvals, but we do consider the party-list setting, where each two projects are either approved by the same voters or by disjoint groups of voters. This assumption most naturally fits the multiwinner framework, where the voters vote for “parties”, but it also makes sense in the PB setting, as it generalizes instances where voters approve exactly one project, which is sometimes enforced in practice, e.g., in Wrocław, Poland (see [12]).

Overall, we find that for most variants of our games Nash equilibria are not guaranteed to exist, and it is computationally challenging to decide if they do. Indeed, this task is often both NP- and coNP-hard, and even finding proposers’ best responses is computationally challenging. However, for the single-project submission setting, our games become much easier to analyze, especially for multiwinner elections. Further, our experiments show that Nash equilibria are common and typically can be found using simple dynamics. Moreover, we demonstrate that the utilities that the proposers obtain under limited information regarding voters’ preferences are similar to those in equilibria based on full profiles. So, in the cases in which the proposers only have access to a reliable polls, equilibria can be computed for small subelections, yielding both quick computation and low demand for precise information about the expected votes.

We emphasize that it is not immediately clear whether the fact that our games are in some contexts hard or easy to play should be considered “negative” or “positive”. Indeed, it is conceivable that the interest of the project proposers (to maximize their profit) might conflict with the interests of the voters or the election designer. We address this issue in Section 6.2.

All missing proofs are available in the full version of the paper [15]. We discuss related work at the end of the paper, in Section 7.

2 PRELIMINARIES

A *participatory budgeting election* (an *election*, for short) is a tuple $E = (C, V, B)$ that consists of a set of *projects* $C = \{c_1, \dots, c_m\}$, a collection of *voters* $V = (v_1, \dots, v_n)$, and a *budget* $B \in \mathbb{N}$. Each project $c \in C$ has a *cost*, denoted as $\text{cost}(c)$, and each voter v has a set of projects $A(v) \subseteq C$ that they approve. For a subset $W \subseteq C$ of projects, we let $\text{cost}(W) = \sum_{c \in W} \text{cost}(c)$ be their total cost. We say that W is *feasible* if $\text{cost}(W) \leq B$. For a project c , by $S(c)$ we mean the set of voters that approve it and we refer to $|S(c)|$ as its *approval score*. A (participatory budgeting) *voting rule* f is a function that given an election outputs a feasible set of projects (note that f is resolute, so we assume an internal tie-breaking mechanism). In the *multiwinner* setting we assume each project to have cost 1.

We will further consider plausible domain restrictions. In the *party-list* setting [5] we assume that for each two projects $a, b \in C$, it holds that either $S(a) = S(b)$ or $S(a) \cap S(b) = \emptyset$. In other words, either two project are supported by the same voters or their sets of supporters are disjoint. Also, the class of *laminar* profiles [29] extends the party-list setting. There, the approval sets of each pair of projects are either disjoint or one of them is a subset of another. Formally, preferences are *laminar* if for each pair of projects $a, b \in C$ either (i) $S(a) \subseteq S(b)$, or (ii) $S(b) \subseteq S(a)$, or (iii) $S(a) \cap S(b) = \emptyset$.

Participatory Budgeting Rules. For the case of PB, we focus on BasicAV, Phragmén [4, 25] and Method of Equal Shares (MES) [28, 29] rules. Each of them starts with an empty set W and performs a number of iterations, either extending W with a single project or dropping some project from consideration. We assume that there is a tie-breaking order \succ over the projects and whenever a rule faces an internal tie, it selects the project ranked highest in this order. For an election $E = (C, V, B)$, our rules work as follows:

BasicAV. In each iteration, BasicAV considers a project c with the highest approval score that it has not considered yet. If $W \cup \{c\}$ is feasible then it includes c in W (i.e., it *selects* or *funds* c) and otherwise it drops it. The rule terminates after considering all the projects.

Phragmén. Initially, the voters have empty bank accounts, but they receive virtual currency at a constant rate of one currency unit per one unit of time. An iteration starts as soon as there is a project c that has not been considered yet, such that voters in $S(c)$ have $\text{cost}(c)$ currency. If $W \cup \{c\}$ is feasible, then Phragmén includes c in W and resets the accounts of the voters from $S(c)$ to zero (these voters *buy* the project); otherwise it drops c . The rule terminates when no further iteration can start.

Method of Equal Shares (MES).² Initially, each voter gets $B/|V|$ virtual currency. Then, each iteration proceeds as follows (let $b(v)$ be the money held by voter v at the beginning of the iteration): For a project c and a number ρ , we say that c

²Several variants of MES have been proposed in the context of participatory budgeting, depending on the presumed utilities of the voters from approved projects. Here we consider the *cost-utility* variant, introduced by Faliszewski et al. [12]. An alternative definition is discussed in the full version of the paper [15].

is ρ -affordable if it holds that:

$$\sum_{v \in S(c)} \min(b(v), \rho \cdot \text{cost}(c)) = \text{cost}(c)$$

i.e., if the voters approving c can afford it, provided that each of them contributes ρ fraction of its cost (or all the money they have left if the ρ fraction would be too much). MES includes in W the project that is affordable for the lowest value of ρ (and takes $\min(b(v), \rho \cdot \text{cost}(c))$ currency from each $v \in S(c)$; these voters *buy* c). MES terminates if no project is affordable for any ρ .

MES is well-known to occasionally output sets of projects that are not exhaustive, i.e., sets that could be extended with some project and remain feasible [28, 29]. In practice, various completion methods are used (see, e.g., the work of Faliszewski et al. [12] for their comparison), but we disregard this issue for simplicity.

For the multiwinner setting, it is also common to study so-called Thiele rules [20, 36] and their sequential variants. This includes, e.g., the approval-based Chamberlin–Courant rule (CC) and the proportional approval voting rule (PAV). We define (both sequential and global) Thiele rules in the full version of the paper [15] and we provide results for them there, but we omit them from the main discussion for the sake of clarity. Indeed, modulo minor technicalities, our results for them are analogous to those for Phragmén or MES.

3 PROJECT SUBMISSION GAMES

Let f be a voting rule. A *project submission game* G for rule f (an f -PSG G) consists of (1) a set of projects $C = \{c_1, \dots, c_m\}$, (2) a set of *project proposers* $P = \{P_1, \dots, P_\ell\}$, (3) a collection of voters $V = (v_1, \dots, v_n)$, and (4) a budget B . The components C, V, B are analogous to a PB election. The proposers are the players in the game and each of them has some potential projects that they can choose to submit or not. Hence, we view $P = \{P_1, \dots, P_\ell\}$ as a partition of C . Thus, we may sometimes speak of a proposer P_i , meaning the player in the game, and of a set of projects P_i , meaning the projects that this proposer may submit. The intention will be clear from the context. We drop the rule f from the notation if it is clear or irrelevant.

Formally, a *strategy* $s_i \subseteq P_i$ of proposer P_i is a nonempty subset of their submitted projects. A *strategy profile* is a collection of such strategies, denoted as $\mathbf{s} = (s_1, \dots, s_\ell)$. By \mathbf{s}_{-i} we denote the strategy profile \mathbf{s} excluding the proposer P_i , and by (\mathbf{s}_{-i}, s'_i) we denote the strategy profile obtained from \mathbf{s} by replacing the strategy of P_i with s'_i . An election *induced* by \mathbf{s} is a tuple $E(\mathbf{s}) = (\bigcup_{i \in [\ell]} s_i, V, B)$. The *utility* of a given proposer P_i is defined as the total cost of the selected projects that they submitted; formally, $u_i(\mathbf{s}) = \text{cost}(f(E(\mathbf{s})) \cap P_i)$.

We analyze the existence, structure, and computational complexity of computing *pure Nash equilibria* (NE) in project submission games. Given a game G , a strategy profile \mathbf{s} is an NE profile if for each proposer P_i and all P_i 's strategies $s'_i \subseteq P_i$, we have $u_i(\mathbf{s}_{-i}, s'_i) \leq u_i(\mathbf{s})$.

Example 3.1. *Take an instance as shown in Section 1. There, we have in total six projects that can potentially enter the elections, i.e., $T_1, T_2, T_3, B_1, B_2, B_3$, who are supported by the numbers of voters as shown in Table 1. Their costs are also as in Table 1, and the budget is € 75 000. Moreover, there are two proposers, T and B , partitioning*

the projects into sets $T = \{T_1, T_2, T_3\}$ and $B = \{B_1, B_2, B_3\}$, and the voting rule used is BasicAV. Consider first the strategy profile $\mathbf{s} = (\{T_1, T_2, T_3, B_1, B_2, B_3\})$. There, we see that the funded projects in the election induced by \mathbf{s} are T_1, B_1, B_2 and B_3 , giving $u_T(\mathbf{s}) = 50000$, and $u_B(\mathbf{s}) = 24000$. Nevertheless, in the profile $\mathbf{s}' = (\{T_2, T_3, B_1, B_2, B_3\})$, $u_T(\mathbf{s}') = 60000$, and $u_B(\mathbf{s}') = 10000$, so \mathbf{s} is not a BasicAV-NE.

In particular, we are interested in computing best responses of given proposers and in checking an NE existence. Formally, we consider the following problems.

Definition 3.1. Let f be a voting rule. (1) In the f -NE EXISTENCE problem we are given an f -PSG G and we ask if there is an NE profile for G . (2) In the f -BEST RESPONSE problem, we are given an f -PSG G , a proposer P_i , a value x , and a strategy profile \mathbf{s}_{-i} of players other than P_i ; we ask if there is a strategy s_i of P_i such that $u_i(\mathbf{s}_{-i}, s_i) \geq x$.

In the next section we analyze the existence of Nash equilibria and the complexity of the above problems for PSGs. Then, in Section 5, we provide similar analysis for a variant of PSGs where each proposer submits exactly one project.

4 THEORETICAL ANALYSIS

Here, we study the conditions under which an NE exists and can be computed efficiently, which turn out to be quite restrictive. Specifically, for each of BasicAV, Phragmén, and MES there are PSGs without Nash equilibria even for party-list profiles (and for BasicAV and MES it even suffices to consider games with a single voter).

Proposition 4.1. *For each rule among BasicAV, Phragmén and MES, there is a PSG with two proposers that does not admit any NE, where the voters have party-list preferences (for the case of BasicAV and MES even with a single voter).*

PROOF. For BasicAV and MES, form a game with two proposers, $P_1 = \{a_1, a_2, a_3\}$ and $P_2 = \{b_1, b_2, b_3\}$ and the projects' costs:

$$\begin{aligned} \text{cost}(a_1) &= 5, & \text{cost}(a_2) &= 4, & \text{cost}(a_3) &= 2, \\ \text{cost}(b_1) &= 6, & \text{cost}(b_2) &= 4, & \text{cost}(b_3) &= 4. \end{aligned}$$

We fix budget $B = 14$ and we let the tie-breaking order be:

$$a_1 \succ b_1 \succ b_2 \succ b_3 \succ a_2 \succ a_3.$$

There is only one voter, approving all the projects, so both rules consider the projects in the tie-breaking order.

First, observe that P_1 always (weakly) benefits from submitting a_2 and a_3 . Indeed, if they are not elected, their presence does not matter. Further, a_3 is the last in tie-breaking, so if it is selected, it does not prevent any other (more valuable) projects of P_1 from being funded. Selecting a_2 could only possibly prevent a_3 from being selected, but $\text{cost}(a_2) \geq \text{cost}(a_3)$. By an analogous reasoning, we can see that P_2 always (weakly) benefits from submitting b_2, b_3 .

Now the only actual dilemma of the players is whether to submit a_1 and b_1 . The utilities of the players from playing the corresponding strategies are summarized in Figure 1. Note that there is no NE in this game (indeed, the cycle of best responses is analogous to the one in the classic *matching pennies* game).

For Phragmén, exactly the same counterexample does not work, as when all the projects have the same support, Phragmén prefers

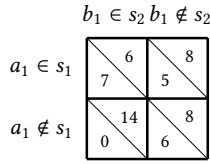


Figure 1: Normal form representation, with rows representing the choices of P_1 and the columns to the choices of P_2 .

cheaper projects (e.g. a_2) to more expensive ones (e.g., a_1). But here it is enough to consider a plurality election in which each project c has a group of $\text{cost}(c)$ supporters. Then under Phragmén all the projects are tied, and the reasoning presented above works. \square

Worse yet, given a project submission game it is intractable to decide if it admits a Nash equilibrium.

Theorem 4.2. *For each rule among BasicAV, Phragmén and MES, NE EXISTENCE is both NP-hard and coNP-hard (for BasicAV and MES this result holds even for party-list profiles with a single voter).*

PROOF. We give an NP-hardness proof for BasicAV and MES. The result for Phragmén will follow from the proof of Theorem 4.3 later on. We reduce from the SUBSET SUM problem, whose instance consists of a set of positive integers U and a target value T ; we ask if there is a subset of U that sums up to T (with every number in U smaller than T). Adapting our reductions, we also obtain coNP-hardness proofs, available in the full version of the paper [15].

Construction: Our reduction forms a PSG with three proposers, P_0, P_1, P_2 . The projects of P_1 and P_2 are as in the proof of Proposition 4.1, whereas P_0 has a single project c_i for each number $x_i \in U$, with $\text{cost}(c_i)$ is the i^{th} number in U . Then, P_0 also has project c^* with cost $15 \cdot T - 14$. Then, $B = 15 \cdot T$. There is a single voter approving all the projects. The tie-breaking order prefers all the projects of P_0 to the projects of P_1 and P_2 , with c^* being least preferred among the projects of P_0 , and between the projects of P_1 and P_2 the tie-breaking is as in the proof of Proposition 4.1. This construction is the same for both voting rules.

Correctness: Suppose that there exists a solution to the considered SUBSET SUM instance. Then, the best strategy for P_0 is to submit the projects corresponding to the choice of elements from U summing up to T . Then the satisfaction of P_0 is $15 \cdot T$ (which is the maximal possible) and the satisfaction of P_1 and P_2 is 0, regardless of their strategies. This strategy profile clearly is an NE.

Suppose now that there does not exist a solution to the considered subset sum problem instance. Then the best strategy of P_0 is to submit only project c^* . Note that any other choice of projects would give P_0 utility at most $15 \cdot T - 15$. However, then there is no NE due to P_1 and P_2 , as shown in the proof of Proposition 4.1. \square

This theorem calls for some comments. First, as we show our problems to be both NP-hard and coNP-hard, we expect their exact complexity to be on the second level of the Polynomial Hierarchy (informally speaking, our hardness proofs put them beyond the first level, and expressing NE existence in the form “there exists a strategy profile such that all deviations are not profitable” puts them within the second level). However, in the follow-up intractability

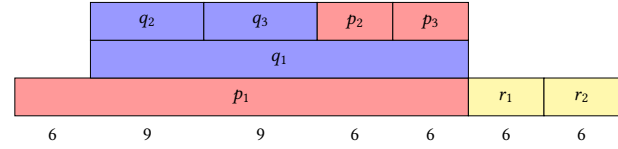


Figure 2: Gadget for the proof of Theorem 4.3. Projects of different proposers are depicted as boxes of different colors. Sets of voters of given cardinalities support projects shown above them (e.g., 9 voters approve set $\{p_1, q_1, q_2\}$).

proofs we generally do not aim to find both NP- and coNP-hardness, or exact completeness results, and focus on the intractability that is more convenient to prove. Second, it is interesting to ask whether Phragmén-NE EXISTENCE also is intractable for party-list profiles. We leave this issue open. Finally, we note that the proof of Theorem 4.2 relies heavily on the projects having different costs. Hence, one might hope for polynomial-time algorithms for the multiwinner setting, where all costs are equal. However, it turns out that this is the case only for BasicAV.

Theorem 4.3. *For the multiwinner setting, we have that:*

- (1) *Each BasicAV-PSG has a polynomial time computable NE.*
- (2) *Phragmén-NE EXISTENCE and MES-NE EXISTENCE are both NP-hard and coNP-hard .*

PROOF. Regarding BasicAV, it is enough to note that in the multiwinner model it boils down to electing B projects with the highest number of approvals (up to tie-breaking). So, it is clear that submitting all the projects is a dominant strategy of every proposer.

We show coNP-hardness of Phragmén-NE EXISTENCE. The constructions for the NP-hardness case and for MES are similar and are presented in the full version. We reduce from the 3-SAT problem, where we are given a propositional logic formula in CNF form (with three literals per clause) and we ask if it is satisfiable.

The Gadget: We first show a Phragmén-PSG for the multiwinner setting that does not admit an NE. There are 48 voters and 3 proposers, $P_1 = \{p_1, p_2, p_3\}$, $P_2 = \{q_1, q_2, q_3\}$, and $P_3 = \{r_1, r_2\}$. All the projects cost 1, and the budget is 4. The voters’ preferences are depicted in Figure 2, with r_1, r_2 being lowest in the tie-breaking order (the remaining part of the tie-breaking order is irrelevant).

Note that P_3 always (weakly) benefits from submitting r_1, r_2 , P_1 (weakly) benefits from submitting p_2, p_3 and P_2 (weakly) benefits from submitting q_2, q_3 (this is so, as for each of the proposers, these are the last projects that Phragmén would—if at all—select for them, and they are approved by disjoint voter sets). Hence, the only actual dilemma is whether P_1 should submit p_1 and if P_2 should submit q_1 . In the full version we show that there is no NE in this game with respect to these two strategies of P_1 and P_2 , and that if we removed two voters supporting p_2 and p_3 respectively, there there would be an NE for P_1 submitting p_1 and P_2 not submitting q_1 .

Construction: Consider a 3-SAT formula φ with k clauses and w variables. We form a Phragmén-PSG as follows. First, we include a copy of the above-described gadget. Second, we add $3w + k + 6$ new projects (each with cost 1): (1) For each variable x , we form two literal projects $c_x, c_{\neg x}$. (2) For each clause, we form a corresponding clause project. (3) We include one extra project c^* . (4) We include

$w + 5$ artificial projects. Tie-breaking is as follows (the exact order within project sets is irrelevant for the proof):

$$\text{literal projects} \succ \text{clause projects} \succ c^* \succ \text{gadget projects} \succ \text{artificial projects}.$$

In addition to the proposers from the gadget, we also have:

- (1) Proposer L , who owns all the literal projects and c^* ,
- (2) Proposer K , who owns all the clause projects, and
- (3) Proposer R , who owns all the artificial projects.

So, we have $48 + 13w + 36$ voters, partitioned as follows:

- (1) We have 48 voters from the gadget, with approval sets as in the gadget, except that one of the voters approving p_2 and one supporter of p_3 also approve all the clause projects.
- (2) For each variable x , we have 7 voters, with 5 of them approving both $c_x, c_{\neg x}$, one voter, called v_x , approving c_x , and one voter, called $v_{\neg x}$, approving $c_{\neg x}$. Additionally, v_x and $v_{\neg x}$ approve all the clause projects that correspond to clauses containing corresponding literals.
- (3) Each artificial project has 6 supporting voters.
- (4) We have 6 voters approving c^* . One of them also approves all the clause projects.

Importantly, each non-gadget project is approved by a group of exactly 6 voters. We set budget $B = w + 5$.

Correctness: First, observe that each artificial project is approved by a disjoint group of voters who do not approve any of the other projects. So, it is always (weakly) profitable for R to submit all of them. Hence, if the game has an NE, it has one where this happens, and we only consider strategy profiles of this form. As there are B artificial projects and their voters can purchase them at moment $1/6$, if some other project becomes affordable at a later time, it is not funded. This means that each non-gadget project c (and also gadget projects p_2, p_3) can only be selected if no other project c' whose set of approving voters intersects with that of c is selected.

From the above observation, we immediately obtain that the following pairs of projects cannot be bought together: (i) any two clause projects, (ii) two literal projects corresponding to the same variable, (iii) any clause project and c^* , (iv) any clause project and either p_2 or p_3 , (v) a clause project and a literal project such that the corresponding literal appears in the corresponding clause.

However, for each pair of literals corresponding to different variables, one is guaranteed to be selected, provided that L submits it (this holds because literal projects are preferred in the tie-breaking order and are supported by six voters each; only p_1, q_1, q_2 and q_3 gadget projects are supported by a larger number of voters).

Suppose that there exists a valuation of variables satisfying φ . Then there is no NE in the constructed game. Indeed, in a strategy maximizing the utility of proposer L , they submit the projects corresponding to the literals satisfying φ , as well as c^* . As literal projects are preferred to the clause ones in tie-breaking, their voters buy them before any of the clause projects can be purchased. Then, as per item (v) above and independently of the strategy of K , no clause project can ever be selected. Then c^* is bought, giving $w + 1$ utility to L (which is the maximal possible utility that L can achieve). Note that if K submitted all their clause projects and L played any other strategy, not corresponding to the valuation satisfying φ , then at least one clause project—corresponding to the not satisfied clause—would be elected, after which c^* would not be elected (hence L gets

only w utility). Finally, there are 4 seats in the committee that we did not discuss yet. Since no clause project is bought, the gadget is separated from the remaining voters, and the analysis in the full version of this paper shows that there is no NE regarding P_1 and P_2 .

Suppose now that the φ is not satisfiable. Then let L submit any w literal projects corresponding to different variables, P_1 submit $\{p_1, p_2, p_3\}$, P_2 submit $\{q_2, q_3\}$ and P_3 , R , and K submit all their projects. Then Phragmén selects successively the projects of P_1 and P_2 after which it selects the literal projects submitted by L , one of the clause projects submitted by K (corresponding to a non-satisfied clause) and the projects of P_3 . Since φ is not satisfiable, it is impossible for L to “block” the election of all the clause projects, which means that it is impossible for c^*, p_2 , and p_3 to be selected. It is then clear that none of the proposers can deviate to improve their utility. Hence, this is an NE. \square

Nevertheless, the multiwinner setting guarantees the existence of equilibria when voters’ preferences are party-list.

Theorem 4.4. *Every Phragmén- and MES-PSG, for the multiwinner setting with party-list preferences, admits a Nash equilibrium.*

PROOF. Let all the proposers submit all their projects. We now show that such a strategy profile \mathbf{s} is an NE. Suppose that it is not the case. Then there exists a proposer P_i who has an incentive to withdraw some of their projects. Suppose that P_i performs such a withdrawal one-by-one. If P_i withdraws a project c that is not selected, the outcome and their utility does not change. However, if P_i withdraws a selected project c , then the only change in the outcome is the removal of c and either (1) adding some other project c' from the same “party” (i.e., a project supported by the same voters), or (2) adding some other project c' from another “party” (only for Phragmén), or (3) electing an outcome that has one project fewer (only for MES). In all three cases, P_i either does not gain anything (if c' is a project of P_i) or loses utility (if c' belongs to another proposer, or the rule selects fewer projects). \square

The requirement for the preferences to be party-list in Theorem 4.4 is quite strict—for example, there exist multiwinner MES- and Phragmén-PSGs with no NE where voters’ preferences belong to a more general class of laminar preferences, as in the case of PSGs used as gadgets in the proof of Theorem 4.3.

In principle, even if testing whether a Nash equilibrium exists is hard, it may still be possible to efficiently compute a best response for a given strategy profile. However, in case of PSGs the tractability of computing best responses behaves in the same way as in the case of computing Nash equilibria (which mostly follows from respective proofs for NE EXISTENCE).

Theorem 4.5. *BEST RESPONSE is (1) NP-hard for BasicAV, Phragmén, and MES, in the general PB model, even with party-list preferences, (2) polynomial-time solvable for BasicAV in the multiwinner model, (3) NP-hard for MES and Phragmén in the multiwinner model, (4) polynomial-time solvable for MES and Phragmén in the multiwinner model with party-list preferences.*

5 SINGLE-PROJECT SUBMISSION GAMES

So far, we have assumed that each proposer submits any nonempty subset of their projects. However, sometimes this is not realistic.

For example, a proposer may have several different variants of a project, such as a bike path that goes through several overlapping routes. Another possibility is that, in a multiwinner election, it may be customary—or enforced by the rules—that a single institution nominates only one candidate. Hence, in this section we focus on games that capture such scenarios.

A *single-project submission game* for rule f (f -PSG/1) is defined in the same way as a regular f -PSG, except that each proposer submits exactly one project. Further, we refer to variants of f -NE EXISTENCE and f -BEST RESPONSE for PSG/1s as, respectively, f -SINGLE-PROJECT NE EXISTENCE and f -SINGLE-PROJECT BEST RESPONSE.

PSG/1s are notably simpler than our full-fledged PSGs. In particular, in the multiwinner setting not only Nash equilibria always exist for all the rules, but also they can be computed in polynomial time. We believe that this setting is quite realistic and, hence, we consider this result important.³

Theorem 5.1. *For BasicAV, Phragmén, and MES every PSG/1 for the multiwinner setting has a polynomial-time computable NE.*

PROOF. Note that all our rules are sequential and satisfy the following condition: If W is the outcome computed in the first i rounds and c is a project not in W , then if we removed c from the election and rerun the rule, the first i rounds would select the same projects as with c present.

Suppose for a while that there is no limit on the number of submitted projects and let the proposers submit all their projects. Consider the first round. Here, project c_1 of some proposer P_1 is selected. By the above observation, we know that c_1 would also be selected if it were the only project submitted by P_1 . As no proposer can obtain utility higher than 1 in a multiwinner PSG/1, submitting c_1 is a dominant strategy for P_1 . Let us then fix the strategy of P_1 as $\{c_1\}$, remove the remaining projects of P_1 from the election and rerun the rule. From our observation, c_1 is still elected in the first round. We now repeat analogous reasoning for projects chosen in further rounds, each time finding a dominant strategy of a new proposer P_i (subject to the fixed choices of the proposers who got their projects selected earlier). In the end, we choose arbitrary legal strategies for the proposers who do not have any projects selected. The presented algorithm returns an NE profile in polynomial time. \square

However, this result does hold in the general PB case.

Proposition 5.2. *For each rule among BasicAV, Phragmén and MES, there is a PSG/1 with two proposers without NE, assuming party-list preferences (for the case of BasicAV and MES even with a single voter).*

Nevertheless, in the multiwinner setting, restricting proposers' actions to submitting only a single project allows for computing best responses in polynomial time, by iterating over all possible strategies of a given player.

Theorem 5.3. *For each rule among BasicAV, Phragmén, and MES, SINGLE-PROJECT BEST RESPONSE is solvable in polynomial time.*

³E.g., in the academic system of the country where some of the authors of this paper come from, every few years there is an election of a Research Excellence Council that decides professors' promotions. For each academic discipline, each university department may nominate a candidate and BasicAV is used to select three council members for this discipline. Consequently, the departments play a PSG/1, as they need to consider the actions of other departments.

While the polynomial-time algorithm from the above theorem is simply a form of brute-force search, it is important to have it. In particular, it allows us to test if a given strategy profile is an NE, and it may help in designing heuristics or dynamics that try to find equilibria when they exist.

6 EMPIRICAL ANALYSIS

Having analyzed the properties of project submission games on the theoretical level, we study the existence of equilibria empirically, focusing on real-life election data. We further provide results on social welfare in Nash equilibria. We conclude with an assessment of how limited information can affect proposers' utility.

6.1 Equilibrium Existence

Our theoretical analysis shows that, in general, Nash equilibria are not guaranteed to exist neither for PSGs nor PSG/1s. In this section we explore how this situation looks in practice.

We considered 187 instances of real-life PB elections with up to 10 projects, taken from the Pabulib dataset [12]. We have chosen this limit on the number of projects due to the computational complexity of dealing with our games. For each instance, we analyzed cases with $\ell \in \{2, 3, 4, 5\}$ proposers, each of whom has a roughly equal number of projects (that is, either $\lfloor m/\ell \rfloor$ or $\lceil m/\ell \rceil$ projects), assigned to them uniformly at random. When the number of projects in an election is smaller than $\ell + 1$, then we omit such an instance. Finally, for each non-discarded instance and number ℓ of proposers, we constructed three PSGs, one for each rule out of BasicAV, Phragmén, and MES. As a result, we obtained 8124 PSGs (it is not simply $187 \cdot 4 \cdot 3$ due to the omitted instances).

The goal of our experiments is to check how typical it is for our games to have equilibria and how difficult they are to compute. Towards this end, for each of our PSGs we executed the following protocol, in which each successive step corresponds to a more involved way of finding an equilibrium: (1) We first check whether the strategy profile in which all the proposers submit all their projects is an NE—henceforth, we refer to such profiles where everyone submits all the projects as *full strategy profiles*;

(2) If the full strategy profile is not an NE, we apply the best-response dynamics: Starting from the full strategy profile, all the proposers simultaneously and independently change their strategies to their best responses. We stop either when an NE is found, or after 10 iterations without reaching an NE (or if we detect a cycle);

(3) If the above dynamics do not end in an NE, then we test if an equilibrium exists using brute-force search. For PSG/1s, the experimental setup is analogous, except there is no clear analog of the full strategy profile. Hence we omit the first step and we use best-response dynamics that starts from a strategy profile chosen uniformly at random. For BasicAV, we present our results in Table 2. The results for other rules are presented in the full version of the paper. Overall, they are similar to the ones for BasicAV.

Somewhat surprisingly, it turns out that not only are Nash equilibria very common in our games, but also they can be computed in a fairly simple way. Specifically, for a vast majority of our PSGs already the full strategy profile is an equilibrium. Our dynamics provide an equilibrium for nearly all of the remaining PSGs. There were only five instances, all of which were Phragmén-PSGs, where

Table 2: NE-existence results for BasicAV-PSGs (top) and BasicAV-PSG/1s (bottom). The column “All” shows the numbers of considered instances. For each instance, we recorded if the respective full strategy profile was an NE (“Full-NE”), if we obtained an equilibrium via the step of our algorithm that applies the best-response dynamics (“BR-NE”), or if there was no equilibrium (“No NE”). We never had to use the brute-force search for BasicAV-PSGs or BasicAV-PSG/1s.

Project Submission Games (PSGs)				
ℓ	All	Full-NE	BR-NE	No NE
2	187	133	48	6
3	179	146	26	7
4	171	149	18	4
5	140	130	9	1

Single-Project Submission Games (PSG/1s)			
ℓ	All	BR-NE	No NE
2	187	177	10
3	179	170	9
4	171	164	7
5	140	136	4

to find an equilibrium we had to resort to the brute-force search. Additionally, for PSG/1s our dynamics always found an equilibrium whenever at least one existed. Notably, we actually never had to exceed 4 iterations of the best response dynamics in order to find an equilibrium. Moreover, out of all the analyzed games in which an equilibrium was found by invoking our dynamics, for 87% of them it was sufficient to run at most one iteration of the dynamics.⁴

Hence, our experiments suggest that simply submitting all the projects is a good basic strategy, but occasionally the proposers may benefit by considering more involved approaches, based on computing their best responses. As for PSG/1s we can compute the best responses in polynomial time, this means that—from practical point of view—finding equilibria for these games is rather easy.

6.2 Voter Satisfaction in Equilibrium

We now analyze how the impact of proposers’ strategic behavior on the satisfaction of the voters under different voting rules. For an outcome $W \subseteq C$, we measure the voters’ satisfaction in two ways:

(1) *Chamberlin-Courant (CC) score*: the number of voters i such that $|A_i \cap W| > 0$, that is, the number of voters who approve at least one selected project. (2) *Social Welfare (SW)*: for each voter i , we define the utility of i from W as $\text{cost}(A_i \cap W)$. Then, SW is defined as the sum of all voters’ utilities: $\sum_{i \in N} \text{cost}(A_i \cap W)$.

Both metrics are standard to the social choice literature, measuring the voters’ satisfaction in an egalitarian and utilitarian way, respectively. We consider the effect that strategizing has on both metrics. Denote as s^* the strategy profile in which all of the proposers nominate all of their projects (which can be viewed as the most natural, ‘honest’ strategy). Then, for a voting rule f and a strategy profile s , we define the *price of deviation* as $\frac{\text{sat}(s) - \text{sat}(s^*)}{\max(\text{sat}(s), \text{sat}(s^*))}$,

⁴In total, we used the dynamics to find an equilibrium in 2242 games. In some PSG/1s games the randomly drawn starting strategy profile was already an equilibrium.

Table 3: Average price of deviation. Each row shows values corresponding to the price of deviation for particular numbers of participating proposers.

ℓ	Phragmén		MES		BasicAV	
	CC	SW	CC	SW	CC	SW
2	-0.02	0.22	-0.12	0.06	-0.18	-0.03
3	0.03	0.23	-0.08	0.04	-0.17	-0.08
4	0.04	0.19	-0.07	0.13	-0.18	-0.03
5	0.02	0.12	-0.12	-0.03	-0.16	-0.06

where $\text{sat}(s)$ is the voters’ satisfaction (measured either as SW or CC score) from the outcome elected by f for s . Note that here, the positive value of a price of deviation indicates that a strategy profile provides higher satisfaction than the natural profile s^* .

For the voting rules we consider, we test the social welfare in an equilibrium and its corresponding price of deviation. Towards this end, we compute equilibria for PSGs based on instances in Pabulib, analogously to the experiment in Section 6.1, and focus only on instances where full strategy profile was not an equilibrium. Then, we consider the average price of deviation for the computed profiles. The results of this experiment are shown in Table 3.

Interestingly, for nearly all the considered rules and numbers of proposers, the average price of deviation measured by the CC score was negative. However, in case of SW, the results are more complex. We observe that in the case of BasicAV, the average price of deviation is negative, which means that reaching an equilibrium proved to be detrimental for the voters in the context of this rule. Nevertheless, in the case of Phragmén, we observe a strong, opposite effect, showing that on average Nash equilibria lead to increased social welfare. In the case of MES, the effect is much smaller, although leaning towards positive price of deviation. Hence, we conclude that the plausible behavior of project proposers has a negative effect in the context of the commonly used BasicAV rule, which is in contrast with more proportional methods.

6.3 Equilibria with Imperfect Information

While in earlier parts of our analysis we assumed that the proposers have full knowledge about the preferences of the voters, it might appear that strategizing under partial information can lead to much different outcomes. To challenge this intuition we empirically check whether choosing strategies based only on a small number of ballots yields a significantly worse utility for the proposers than in the situation in which voters’ preferences are fully known.

In this experiment, we consider all of those PSG instances analyzed previously in which there is an NE; as shown before, this is the case for nearly all of the previously-considered instances. For each instance, we execute the following scheme:

(1) For each fraction $\alpha \in \{1\%, 5\%, 10\%, 25\%, 50\%\}$, we sample 20 subelections in which only preferences of the α of all voters are known. Subsequently, we compute Nash equilibria for games based on those subelections; we discard the rare instances with no NE.

(2) For each subelection, we check the absolute difference between the utility of a proposer in the NE for the subelection (from the perspective of the full PSG) and in the NE for the full instance,

Table 4: Imperfect-information effect for BasicAV. Presented are the average maximum normalized (absolute) differences of the utilities of proposers in an NE for the full instance and for a subelection; shown separately for a range of 2 to 5 proposers.

ℓ	1%	5%	10%	25%	50%
2	0.11	0.10	0.07	0.05	0.04
3	0.21	0.20	0.16	0.14	0.10
4	0.11	0.09	0.09	0.07	0.05
5	0.19	0.19	0.14	0.10	0.04

and then normalize the difference by the latter utility. We record the maximum of such a normalized difference over all proposers.

Surprisingly, we observe that the differences in proposers’ utilities are limited. Table 4 depicts the results for BasicAV. Indeed, as the number of known ballots increases, the average maximum normalized (absolute) differences drop below 20%. Crucially, for a rather small number of participants in PB elections, it is plausible for the proposers to learn preferences of a high fraction of voters.

Those results are strengthened by the following complementary experiments. First, as Table 5 shows, for the case of BasicAV, in a vast majority of instances the utility of proposers was not altered at all due to limited information.⁵ Second, we checked that playing according to recommendations obtained for partial knowledge is almost universally better (or as good as) ignoring the issue and submitting all the projects that a proposer has, even for 1% samples. While we focused on BasicAV, the results for the remaining rules presented in the full version of the paper [15] are qualitatively very similar. Hence, we argue that in the case of the games we consider, the equilibria computed under the assumption of full knowledge remain close to those that can be achieved with limited information.

7 RELATED WORK

We conclude our discussion by describing related work. Our study is closely related to the line of work on *strategic candidacy*, initiated by Dutta et al. [9] and continued, e.g., by Eraslan and McLennan [10], Rodríguez-Álvarez [33, 34], Lang et al. [22], Polukarov et al. [30], and Brill and Conitzer [3]. In a candidacy game, the candidates have preferences on election outcomes and want to obtain as favorable result as possible by choosing to either run or withdraw from the race. The main issues regard existence, nature, and complexity of computing Nash equilibria in such games. While most of this literature focuses on the single-winner case, Obraztsova et al. [27] considered the multiwinner setting. The main difference between PSGs and candidacy games is that in the former the proposers derive value only from their projects, whereas in the latter the candidates care about the performance of the other candidates.

Nominating candidates in an election was also studied by Faliszewski et al. [13], who asked for the complexity of deciding if a party can nominate an election winner in a nonstrategic setting. Their work focusing on the simple Plurality rule was later extended by Cechlárová et al. [7] who studied more involved rules. While the setting here is very related to ours, the difference is that the

⁵In fact, in almost all of those cases proposers submitted all of the projects.

Table 5: Percentage of instances for BasicAV with no differences between maximum normalized (absolute) differences of the utilities in an NE for subelections and that for the full instances, averaged over subelections; rounded to integers.

ℓ	1%	5%	10%	25%	50%
2	47%	55%	62%	71%	77%
3	51%	58%	64%	73%	82%
4	58%	63%	70%	77%	85%
5	66%	71%	80%	82%	87%

authors focus on single-winner ordinal elections, fix the submission limit, and ask for the possible (or necessary) winners rather than for the existence of Nash equilibria. Nonetheless, game-theoretic analysis of related settings was covered, e.g., by Harrenstein and Turrini [19]. Also, the study of finding equilibria in nomination games has been explored in several models related to voting, e.g., the Hotelling-Downs model [8, 18, 26] or tournaments [23, 24].

In the context of participatory budgeting, our work is related to that of Sreedurga [35] and Faliszewski et al. [14], both of which consider the setting where each project has several different variants, each with a different cost. Further, shortlisting of projects has also been studied by Rey et al. [2021]. Sreedurga [35] studies computational and normative properties of voting rules that take different project variants as explicit part of the input, whereas Faliszewski et al. [14] consider Nash equilibria in games where project proposers choose the costs (and want to maximize the amount of funds they can get). These games are similar to ours for the case of single project submission games, except that Faliszewski et al. [14] assume that the costs come from a continuous interval and do not affect voters’ approvals, whereas we only consider a discrete set of project variants, each with a possibly different support. Hence, our work concerns a significantly different type of proposers’ actions.

8 CONCLUSION

We initiated the study of strategic behavior of project proposers in participatory budgeting and multiwinner voting. While we showed intractability of checking the existence of an NE in many settings, we also found some tractable cases, of which the most important is that of multiwinner voting, where each proposer nominates a single candidate. Further, our experiments show that in practice equilibria are common and often have a simple structure.

Our results open a large room for future work. In particular, it would be interesting to extend the study of study games where proposers have limited knowledge about the votes and/or competing proposers. While our experimental results make a step towards this goal, theoretical analysis of PSGs with limited information remains to be addressed, e.g., by incorporating uncertainty about voters’ preferences or potential projects that other proposers can put forward. Furthermore, while pure strategy Nash equilibrium is an appealing concept, the study of PSGs would benefit from the study of more elaborate ones, such as trembling hand equilibria. Furthermore, it is of natural interest whether the intractability of computational problems we consider can be mitigated, e.g., through parametrized algorithms, with most natural being the number of candidates or the number of voters.

ACKNOWLEDGMENTS

We are grateful to AAMAS-26 anonymous reviewers for their insightful feedback. This project has received funding from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation programme (grant agreement No 101002854). Views and opinions expressed are however those of the authors only and do not necessarily reflect those of the European Union or the European Research Council. Neither the European Union nor the granting authority can be held responsible for them. This work was co-funded by the European Union under the project Robotics and advanced industrial production (reg. no. CZ.02.01.01/00/22_008/0004590). Grzegorz Lisowski acknowledges support by the European Union under the Horizon Europe project Perycles (Participatory Democracy that Scales). The work has been supported in part by the AI2050 program at Schmidt Sciences (Grant G-24-66104). Grzegorz Pierczyński was supported by Poland’s National Science Center grant no. 2022/45/N/ST6/00271.

Andrzej Kaczmarczyk worked on the project in part during his employment at AGH University and The University of Chicago. Grzegorz Lisowski has been employed at AGH University while working on this paper.



Funded by
the European Union



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