Positional Properties in Temporal Logic

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ABSTRACT

Non-terminating systems are often modelled with the use of temporal logic: ATL* and Strategy Logic are two logics designed for multi-agent systems in particular. Their semantics are based on the existence of strategies that enable groups of agents to enforce specified goals. It has been noted that for certain fragments of ATL*, the semantics are equivalent even when we restrict to positional strategies, which is beneficial as positional strategies yield favourable algorithmic properties for model-checking and strategy synthesis. However, there has not been much study of the necessary and sufficient conditions for a fragment of temporal logic to have equivalent semantics under positional and memoryful strategies - most existing work on positional strategies in infinite games assumes observations are on edges rather than states, a distinction that can affect whether a property is positional. We investigate this problem, as well as a similar phenomenon noted in Strategy Logic; in certain fragments, existentially quantified strategies do not depend on entire universally quantified strategies in their scope, but only on moves chosen by those strategies at the current state.

KEYWORDS

Temporal Logic; ATL; Strategic Reasoning; Positional Strategies

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1 INTRODUCTION

Temporal logics such as CTL* and ATL* distinguish between state formulae, evaluated at states, and path formulae, evaluated over infinite paths and used to express temporal properties. The satisfaction of a formula is defined by the existence of a strategy which enforces the temporal property in question. Such logics can often be intractable, or have unrealistic assumptions about the abilities of real-world agents. Therefore, there is a body of work on the effect of limiting the resources available to agents in temporal logics. One example of such a limitation is the memory an agent is allowed to use in their strategy. For example, has been found that for games of imperfect information, the semantics differ under infinite-memory strategies and finite-memory strategies [22]. This contrasts with the case of perfect information where finite-memory strategies suffice. It has been noted for some fragments of ATL^* that the semantics could equivalently be defined using *positional* strategies, which prescribe an action to take at each state (rather than the whole history of states seen so far). The canonical example of this is ATL [4], which only allows the path formulae $X\varphi$, $\varphi_1 U \varphi_2$ and $\varphi_1 R \varphi_2$.

Strategy Logic, which allows for arbitrary quantification of strategies that can then be bound to certain agents, is much more expressive than ATL*, but is known to have highly intractable model-checking and an undecidable satisfiability problem [17, 18]. Therefore there has been interest in finding tractable fragments of Strategy Logic. One way this has been achieved is by defining a 'behavioural' semantics, which restricts the ways in which an agent's strategy is able to depend on another agent's strategy [19]. As well as tractability, the kind of dependence admitted by a fragment of strategy logic affects model-theoretic properties it has and the semantics we can set up for it, so the question of what we can express under a certain form of dependence is an interesting one.

Typically the limitation on these resources is motivated by some conceptual problem, and problems such as model-checking and strategy synthesis are often more tractable in these frameworks, as the space of strategies is smaller. It would therefore be interesting to know when we can apply these more tractable resource-limited frameworks without compromising our reasoning; which formulae have the same behaviour in the standard and resource-limited frameworks? And given a tractable fragment of a temporal logic, can we isolate the behaviour that makes this tractable? We focus on the specific cases of memory limitations on player strategies, and restricted forms of dependence in Strategy Logic.

2 POSITIONAL PROPERTIES

A set of infinite words is positional if, when taken as the winning condition for any game over a graph, player 1 has a winning positional strategy whenever they have a winning memoryful strategy. The positionality of objectives in infinite games has been long studied, with perhaps the most well-known result being the finding that parity games are positionally determined [9]. Full characterisations of languages positional for both players have existed for a while [11], but it has taken longer to break ground on positionality for a single player. Recently, a number of full characterisations of positional ω-regular languages have been presented [5], and a characterisation has been found for half-positional languages in general [21]. It has also been found that ω-regular languages are exactly the objectives which can be played optimally with finite memory in infinite graphs [3]. However, many of these existing results are based on games where observations lie on edges, whereas in temporal logic observations lie on states. It is possible for a property

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to be positional over state-labelled models but not edge-labelled models [14]. There is, however, a link between the memory requirements of state-labelled graphs and edge-labelled graphs. We have found that properties positional over state-labelled models require at most knowledge of the previous observation to play optimally in edge-labelled models.

It is known that for LTL model checking, we only need to check the ultimately periodic traces of exponential size relative to the formula [7] - if a formula is positional, we can restrict further the class of ultimately periodic traces we must check, yielding improved complexity bounds. This also applies to most temporal logics which use LTL path formulae. It is well-known that LTL path formulae express only ($\omega-$)star-free languages [8]. It turns out that this is sufficient to express all positional ω-regular languages.

Theorem 2.1. *All ω-regular positional languages are ω-star-free.*

Aligning the study of positional languages with the algebraic techniques typically used to study star-free languages has also lead to some nice necessary and sufficient conditions for positionality over certain subclasses of games. It is also simple to check these conditions hold given a Wilke algebra recognising a language.

It is known that all prefix-independent bipositional languages can be expressed as the winning condition for a finite-colour parity game [6], and that all half-positional Muller conditions can be expressed as Rabin conditions [23]. The existence of paths satisfying these formula can both be expressed in the logic CTL extended with the state formula $\exists(GF\varphi_1 \wedge FG\varphi_2)$. It turns out this fragment is subsumed by the strictly more expressive CTL extended with state formula $\exists G(\varphi_1 U \varphi_2)$. In both of these extensions to CTL, all path formulae are positional, providing some reasonably expressive positional fragments.

There is not currently a full characterisation of the ω-regular prefix-independent positional languages[1], although it seems to be well-behaved enough to potentially be captured as a syntactic fragment of LTL. Whilst Rabin conditions are in this class, they are not the only such languages; we have found prefix-independent path formulae which cannot be encoded as a Rabin condition.

It should be noted that a fragment of e.g. ATL* differing on positional and memoryful semantics does not preclude finding an equally expressive positional fragment. For example, ATL^+ can express path formulae which are not positional. However, it is equally as expressive as ATL [13], so for any ATL^+ formula we can find an equivalent one which only contains positional path formulae.

3 STRATEGY DEPENDENCY

It is known for fragments of Strategy Logic that admitting *behavioural* dependence leads to good model-theoretic properties. We would usually expect an existentially quantified strategy to depend on universally quantified strategies it is in the scope of - it can ‘see’ what the universally quantified strategies are selecting on every path, including those in the future or along counterfactual plays. If a fragment is behavioural, this means that we can define strategies pointwise on paths; given a history π, the action of a strategy $\sigma(\pi)$ should only depend on the moves chosen by universally quantified strategies along the path π [16]. We can see this as allowing for restricted forms of a kind of Skolemisation - again, we are limiting the resources of a given player, in this case the information they have on their opponent’s strategy. Currently, it is known that allowing only disjunctive or only conjunctive combinations of path formulae permits behavioural dependence, but that allowing Boolean combinations of path formulae violates this property [19].

This approach of finding simpler ways of ‘Skolemising’ certain fragments of Strategy Logic was continued in [10], which finds a fragment where strategies depend on moves chosen by other agents’ strategies along the history current path, but where it can depend on moves made by *any* strategy along the current history and not just universally quantified strategies it is in the scope of. In [2] various semantics are presented to capture independence-friendly quantifiers in Strategy Logic, where existential quantifiers do not have to depend on all universal quantifiers they are in the scope of.

The fragment of Strategy Logic $SL^-[SG]$ [1], which allows for blocks of alternating quantifiers followed by a single temporal operator, can also be expressed with a simpler form a strategic dependence. We have found this fragment allows us to define strategies pointwise on states; the semantics are equivalent even if we allow only positional strategies, and have it such that the action of a strategy at a state q depends only on the actions of other in-scope strategies at q. A useful aspect of this state-local dependence is that we can treat each state its own discrete game. This allows us to use the machinery of effectivity functions to cast concurrent game models into the more well-behaved neighbourhood models [20].

4 FUTURE WORK

It seems unlikely we could find a syntactic fragment of LTL which expresses *all* ω-regular positional properties. Positionality is quite a non-local property, and especially does not compose well with respect to the usual operators in LTL. Most damningly, two positional properties ψ_1, ψ_2 can be such that $\psi_1 \vee \psi_2$ is *not* positional. For the latter, the disjunction of two ($\omega-$regular) positional properties is positional only when the resulting language has a totally ordered set of residuals. However, certain restricted classes of positional properties, such as prefix-independent ω-regular languages have much better compositional properties and may correspond to a more natural fragment of LTL.

There has been some precedent for the idea of approximating model-checking of complex formulae ϕ with simpler formulae ψ_1, ψ_2 encoding an upper and lower bound in the sense that $\psi_1 \rightarrow \phi \rightarrow \psi_2$ [13]. There is some hope of achieving this with positional properties also, taking advantage of the efficient model checking. The most promising method would most likely be through a sequence of syntactic transformations to a fragment of ATL* known to be positional.

We also aim to find larger fragments of Strategy Logic with state-based or behavioural dependence, as this should allow the semantics to be defined over effectivity models such as those of [12]. The hope is that this will provide a unifying way of handling model-theoretic constructions across these fragments, which would allow e.g. methods of proving completeness for a certain fragment to be re-used across others.

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[1] There exist non-ω-regular prefix-independent positional languages, e.g. certain energy games [15]

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